unidad 2

1. Desarrola y simplifica las siguientes operaciones con potencias:

a) $2^3 \cdot 2^6 \cdot 8^3 = 2^3 \cdot 2^6 \cdot \underbrace{(2^3)^3}_{3 \times 3 = 9} = 2^3 \cdot 2^6 \cdot 2^9 =$

$= 2^{3+6+9} = 2^{18}$

$8 = 2 \cdot 2 \cdot 2 = 2^3$

solución: 2^{18}

b) $(-3)^4 \cdot (-3)^7 \cdot ((-3)^2)^3 = (-3)^{17}$

$(-3)^4 = 3^4 \rightarrow$ al tratarse de una potencia par

$((-3)^2)^3 = (-3)^{2\times 3} = (-3)^6 = 3^6$

$\underbrace{3^4 \cdot 3^6}_{4+6=10} \cdot (-3)^7 = \underbrace{3^{10} \cdot (-3)^7}_{+\cdot - = -} = (-3)^{10+7} = (-3)^{17}$

solución: $(-3)^{17}$

c) $5^3 \cdot 5^{\frac{1}{3}} \cdot 5^{-3} = \cancel{5^3} \cdot 5^{\frac{1}{3}} \cdot \frac{1}{\cancel{5^3}} =$

$5^{\frac{1}{3}} = \sqrt[3]{5}$

$5^{-3} = \frac{1}{5^3}$

solución: $\sqrt[3]{5} = 5^{\frac{1}{3}}$

d) $\frac{2^6 \cdot 2^4 \cdot 2^{-3}}{2 \cdot 2^3} = \frac{2^{6+4} \cdot \frac{1}{2^3}}{2^{1+3}} = \frac{2^{10}}{2^4 \cdot 2^3} =$

$= \frac{2^{10}}{2^{4+3}} = \frac{2^{10}}{2^7} = 2^{10-7} = 2^3$

$2^{-3} = \frac{1}{2^3}$

solución: 2^3

e) $\left(\frac{3}{5}\right)^3 \cdot \left(-\frac{3}{5}\right)^4 \cdot 5^7 = \left(\frac{3}{5}\right)^3 \cdot \left(\frac{3}{5}\right)^4 \cdot 5^7 =$

$= \left(\frac{3}{5}\right)^{3+4} \cdot 5^7 = \left(-\frac{3}{5}\right)^4 = \left(\frac{3}{5}\right)^4$ potencia par, se transforma en positivo

$= \left(\frac{3}{5}\right)^7 \cdot 5^7 = \frac{3^7}{\cancel{5^7}} \cdot \cancel{5^7} = 3^7$

solución: 3^7

f) $\left(\frac{8}{24}\right)^3 : \left[\left(\frac{28}{35}\right)^3 \cdot \left(\frac{25}{343}\right)^2\right]^2 =$

$8 = 2^3$ | $28 = 2^2 \cdot 7$ | $25 = 5^2$

$24 = 2^3 \cdot 3$ | $35 = 5 \cdot 7$ | $343 = 7^3$

$= \left(\frac{\cancel{2^3}}{\cancel{2^3} \cdot 3}\right)^3 : \left[\left(\frac{2^2 \cdot \cancel{7}}{5 \cdot \cancel{7}}\right)^3 \cdot \left(\frac{5^2}{7^3}\right)^2\right]^2 =$

$= \left(\frac{1}{3}\right)^3 : \left[\left(\frac{2^2}{5}\right)^3 \cdot \left(\frac{5^2}{7^3}\right)^2\right] =$

$= \left(\frac{1}{3}\right)^3 : \left[\left(\frac{2^{2\cdot 3}}{5^{1\cdot 3}}\right) \cdot \left(\frac{5^{2\cdot 2}}{7^{3\cdot 2}}\right)\right]^2 =$

$= \left(\frac{1}{3}\right)^3 : \left[\left(\frac{2^6}{\cancel{5^3}}\right) \cdot \left(\frac{\cancel{5^4}}{7^6}\right)\right]^2 =$

$= \left(\frac{1}{3}\right)^3 : \left[\frac{2^6 \cdot 5}{7^6}\right] = \left(\frac{1}{3}\right)^3 : \left[\frac{2^{6\cdot 2} \cdot 5^{1\cdot 2}}{7^{6\cdot 2}}\right] =$

$= \left(\frac{1}{3}\right)^3 : \left[\frac{2^{12} \cdot 5^2}{7^{12}}\right] : \frac{1^3}{3^3} : \frac{2^{12} \cdot 5^2}{7^{12}} =$

$$= \frac{1^3 \cdot 7^{12}}{3^3 \cdot 2^{12} \cdot 5^2} = \frac{7^{12}}{3^3 \cdot 2^{12} \cdot 5^2} =$$

$$= \frac{13841287201}{2764800}$$

solución: $\frac{7^{12}}{3^3 \cdot 2^{12} \cdot 5^2}$

g) $$\frac{12^3 \cdot (-8^4)^2 \cdot 2^3}{2^7 \cdot (-24^2)^3} = \frac{(2^2 \cdot 3)^3 \cdot ((-2^3)^4)^2 \cdot 2^3}{2^7 \cdot \underbrace{(-24)^6}} =$$

$$= \frac{2^{2\cdot 3} \cdot 3^3 \cdot (-2)^{3\cdot 4\cdot 2} \cdot 2^3}{2^7 \cdot (2^3 \cdot 3)^6} =$$

potencia par = resultado positivo

$$= \frac{2^6 \cdot 3^3 \cdot (-2)^{24} \cdot 2^3}{2^7 \cdot 2^{3\cdot 6} \cdot 3^6} = \frac{2^6 \cdot 3^3 \cdot 2^{24} \cdot 2^3}{2^7 \cdot 2^{18} \cdot 3^6} =$$

$$= \frac{2^{33} \cdot 3^3}{2^{25} \cdot 3^6} = \frac{2^8}{3^3} = \frac{256}{27}$$

$$2^{33} : 2^{25} = 2^{33-25} = 2^8$$

$$3^3 : 3^6 = 3^{3-6} = 3^{-3} = \frac{1}{3^3}$$

2. Extrae factores y simplifica al máximo los siguientes radicales:

a) $$\sqrt{8a^4b^7c^3} = \sqrt{2^3a^4b^7c^3} =$$

$$= 2a^2b^3c\sqrt{2bc}$$

radicando = 2

$8 = 2^3$

$2^3 \rightarrow 3 \,\lfloor 2$; $1 \quad 1$

sale fuera

se queda en la raíz

$a^4 \rightarrow 4 \,\lfloor 2$; $0 \quad 2$

$b^7 \rightarrow 7 \,\lfloor 2$; $1 \quad 3$

$c^3 \rightarrow 3 \,\lfloor 2$; $1 \quad 1$

solución: $2a^2b^3c\sqrt{2bc}$

b) $$\sqrt[3]{\frac{1296 \cdot c^9}{243 \cdot a^4 \cdot b^5}} = \sqrt[3]{\frac{2^4 \cdot 3^4 \cdot c^9}{3^5 \cdot a^4 \cdot b^5}} =$$

$$\frac{2 \cdot 3 \cdot c^3}{3ab}\sqrt[3]{\frac{2 \cdot 3}{3^2ab^2}} =$$

1296	2
648	2
324	2
162	2
81	3
27	3
9	3
3	3
1	

$1296 = 2^4 \cdot 3^4$

$243 = 3^5$

243	3
81	3
27	3
9	3
3	3
1	

$2^4 \rightarrow 4 \,\lfloor 3$; $1 \quad 1$ $\qquad c^9 \rightarrow 9 \,\lfloor 3$; $0 \quad 3$

$3^4 \rightarrow 4 \,\lfloor 3$; $1 \quad 1$

dentro fuera

$3^5 \rightarrow 5 \,\lfloor 3$; $2 \quad 1$

$$= \frac{2 \cdot \cancel{3} \cdot c^3}{\cancel{3}ab}\sqrt[3]{\frac{2 \cdot \cancel{3}}{3^{\cancel{2}}ab^2}} = \frac{2c^3}{ab}\sqrt[3]{\frac{2}{3ab^2}}$$

solución: $\frac{2c^3}{ab}\sqrt[3]{\frac{2}{3ab^2}}$

c) $$\sqrt{\sqrt{\sqrt{\left(\frac{6x^5b^2c^8}{75x^2y^3}\right)^3 \cdot \left(\frac{25b^7}{16^8 \cdot x^{-3}c^7}\right)^2}}} =$$

$\sqrt{\sqrt{\sqrt{\ }}} = \sqrt[8]{\ }$

$2^3 = 8$

$$= \sqrt[8]{\left(\frac{6x^5b^2c^8}{75x^2y^3}\right)^3 \cdot \left(\frac{25b^7}{16^8 \cdot x^{-3}c^7}\right)^2}$$

$\sqrt{\sqrt{\sqrt{\ }}} = \sqrt[8]{\ }$

$2^3 = 8$

$$x \rightarrow \frac{2 \cdot \cancel{3}x^{\cancel{5}\,3}b^2c^8}{\cancel{3} \cdot 5^2 \cdot \cancel{x^2}y^3} = \frac{2 \cdot x^3b^2c^8}{5^2y^3}$$

$$\frac{5^2b^7}{(2^4)^8\,\frac{1}{x^3} \cdot c^7} = \frac{5^2b^7x^3}{2^{32} \cdot c^7}$$

$\div$

$$\left(\frac{2 \cdot x^3 \cdot b^2 \cdot c^8}{5^2 \cdot y^3}\right)^3 = \frac{2^3 \cdot x^9 \cdot b^6 \cdot c^{24}}{5^6 \cdot y^9}$$

$$\underbrace{\left(\frac{5^2 \cdot b^7 \cdot x^3}{2^{32} \cdot c^7}\right)^2 = \frac{5^4b^{14}x^6}{2^{64} \cdot c^{14}}}_{x}$$

$$\frac{2^3\cdot x^9\cdot b^6\cdot c^{24}\cdot 5^4\cdot b^{14}x^6}{5^6\,y^9\,2^{64}\,c^{14}} =$$

$$= \frac{\cancel{2^3}\,x^{15}b^{20}c^{\cancel{24}^{10}}\,\cancel{5^4}}{5^{6^2}\,y^9\,2^{\cancel{64}^{61}}\,\cancel{c^{14}}} = \frac{x^{15}b^{20}c^{10}}{5^2\,y^9\,2^{61}}$$

$$\sqrt[8]{\frac{x^{15}b^{20}c^{10}}{5^2\,y^9\,2^{61}}} = \frac{xb^2c}{y2^7}\sqrt[8]{\frac{x^7b^4c^2}{5^2\,y2^5}} =$$

$$\frac{xcb^2}{128y}\sqrt[8]{\frac{c^2b^4x^7}{800y}}$$

$$\begin{array}{r|l} 15 & 8 \\ \hline 7 & 1 \end{array} \qquad \begin{array}{r|l} 61 & 8 \\ \hline 5 & 7 \end{array}$$

solución: $\dfrac{xcb^2}{128y}\sqrt[8]{\dfrac{c^2b^4x^7}{800y}}$

$$\begin{array}{r|l} 20 & 8 \\ \hline 4 & 2 \end{array} \qquad \begin{array}{r|l} 10 & 8 \\ \hline 2 & 1 \end{array} \qquad \begin{array}{r|l} 9 & 8 \\ \hline 1 & 1 \end{array}$$

d) $$\left[\sqrt[12]{\left(a^5b^9\sqrt[4]{a^8}\right)^6}\right]^3 = \left[\sqrt[12]{\left(a^5b^9a^2\right)^6}\right]^3 =$$

$$\sqrt[4]{\left(a^5b^9a^2\right)^6} = \sqrt[4]{a^8} = a^{\frac{8}{4}} = a^2$$

$$\left[\sqrt[12]{\ }\right]^3 \Rightarrow 12:3=4$$

$$= \sqrt[4]{\left(a^7b^9\right)^6} = \sqrt[4]{\left(a^{42}b^{54}\right)} = a^{10}b^{13}\sqrt[4]{a^2b^2}$$

$$a^{7\cdot 6} = a^{42} \longrightarrow \begin{array}{r|l} 42 & 4 \\ \hline 02 & 10 \end{array}$$

$$b^{9\cdot 6} = b^{54} \longrightarrow \begin{array}{r|l} 54 & 4 \\ \hline 14 & 13 \\ 2 & \end{array}$$

solución: $a^{10}b^{13}\sqrt[4]{a^2b^2}$

3. Introduce todos los valores dentro de los radicales expuestos a continuación, simplificándolos posteriormente:

a) $$\frac{8ab^3}{4c^2}\cdot\sqrt{\frac{c^3}{243\cdot a^3}} = \sqrt{\frac{8^2\cdot a^2\cdot (b^3)^2\cdot c^3}{243\cdot a^3\cdot 4^2\cdot (c^2)^2}} =$$

$$8\sqrt{\ } \Rightarrow \sqrt{8^2}$$

$$= \sqrt{\frac{(2^3)^2a^2b^6c^3}{3^5a^3(2^2)^2c^4}} = \sqrt{\frac{2^{\cancel{6}^2}\,\cancel{a^2}\,b^6\,\cancel{c^3}}{3^5\,a^{\cancel{3}}\,\cancel{2^4}\,c^{\cancel{4}}}} = \sqrt{\frac{2^2b^6}{3^5ac}} =$$

$$= \sqrt{\frac{4b^6}{243ac}}$$

solución: $\sqrt{\dfrac{4b^6}{243ac}}$

b) $$\frac{3^4a^2}{25b^3}\cdot\sqrt[3]{\frac{1000b^2}{2^{12}a^3}} = \sqrt[3]{\frac{2^35^3b^2(3^4)^3(a^2)^3}{2^{12}a^3(5^2)^3(b^3)^3}} =$$

$$1000 = 2^3\cdot 5^3$$

$$\sqrt[3]{\frac{\cancel{2^3}\,\cancel{5^3}\,\cancel{b^2}\,3^{12}a^{\cancel{6}3}}{2^{\cancel{12}9}\,\cancel{a^3}\,5^{\cancel{6}3}\,b^{\cancel{9}7}}} = \sqrt[3]{\frac{3^{12}\cdot a^3}{2^95^3b^7}} =$$

$$= \sqrt[3]{\frac{531441a^3}{64000b^7}}$$

solución: $\sqrt[3]{\dfrac{3^{12}\cdot a^3}{2^95^3b^7}} = \sqrt[3]{\dfrac{531441a^3}{64000b^7}}$

c) $$\frac{(7^2+15^2)^3}{(a+b^3)^2}\cdot\sqrt[4]{\frac{a^3-b^3}{5^7}} =$$

$$= \frac{(274)^3}{a^2+b^6+2ab^3} = \sqrt[4]{\frac{a^3-b^3}{5^7}} =$$

$$\left.\begin{array}{l} 7^2 = 49 \\ 15^2 = 225 \end{array}\right\} 225+49 = 274$$

$$(a+b^3)^2 = a^2+(b^3)^2+2ab^3$$

$$= \sqrt[4]{\frac{\left((274)^3\right)^4\cdot(a^3-b^3)}{(a^2+b^6+2ab^3)^4\cdot 5^7}}$$

solución: $\sqrt[4]{\dfrac{\left((274)^3\right)^4\cdot(a^3-b^3)}{(a^2+b^6+2ab^3)^4\cdot 5^7}}$

4. Realiza las operaciones después de haber extraído el máximo de factores posible:

a) $$\sqrt{600}+\sqrt{1800}-\sqrt{576}+\sqrt{72}-\sqrt{1024} =$$

$$\sqrt{2^3\cdot 3\cdot 5^2}+\sqrt{2^33^25^2}-\sqrt{2^6\cdot 3^2}+\sqrt{2^3\cdot 3^2}-\sqrt{2^{10}} =$$

$$= 2\cdot 5\sqrt{2\cdot 3}+2\cdot 3\cdot 5\sqrt{2}-2^3\cdot 3+2\cdot 3\sqrt{2}-2^5 =$$

$$= 10\sqrt{6}+30\sqrt{2}-24+6\sqrt{2}-32 =$$

$$= -56+36\sqrt{2}+10\sqrt{6}$$

$$600 = 2^3\cdot 3\cdot 5^2$$

$$1800 = 2^3\cdot 3^2\cdot 5^2$$

$$576 = 2^6\cdot 3^2$$

$$72 = 2^3\cdot 3^2$$

$$1024 = 2^{10}$$

solución: $-56+36\sqrt{2}+10\sqrt{6}$

b) $3\sqrt{96}+4\sqrt{216}+3\sqrt{27}-6\sqrt{2187}=$

$=3\sqrt{2^5\cdot 3}+4\sqrt{2^3\cdot 3^3}+3\sqrt{3^3}-6\sqrt{3^7}=$

$=3\cdot 2^2\sqrt{2\cdot 3}+4\cdot 2\cdot 3\sqrt{2\cdot 3}+3\cdot 3\sqrt{3}-6\cdot 3^3\sqrt{3}=$

$=\underbrace{12\sqrt{6}+24\sqrt{6}}+\underbrace{9\sqrt{3}-162\sqrt{3}}=$

$=36\sqrt{6}-153\sqrt{3}$

$96=2^5\cdot 3$

$216=2^3\cdot 3^3$

$27=3^3$

$2187=3^7$

c) $\sqrt{700}+2\sqrt{567}-10\sqrt{350}=$

$=\sqrt{2^2\cdot 5^2\cdot 7}+2\sqrt{3^4\cdot 7}-10\sqrt{2\cdot 5^2\cdot 7}=$

$=2\cdot 5\sqrt{7}+2\cdot 3^2\sqrt{7}-10\cdot 5\cdot\sqrt{2\cdot 7}=$

$=10\sqrt{7}+18\sqrt{7}-50\sqrt{14}=$

$=28\sqrt{7}-50\sqrt{14}$

$700=2^2\cdot 5^2\cdot 7$

$567=3^4\cdot 7$

$350=2\cdot 5^2\cdot 7$

solución: $28\sqrt{7}-50\sqrt{14}$

d) $3\sqrt{8x^3}-2\sqrt{144x^3}+\dfrac{3x^2}{2}\sqrt{\dfrac{162x^4}{x^5}}=$

$=3\sqrt{2^3x^3}-2\sqrt{2^4\cdot 3^2\cdot x^3}+\dfrac{3x^2}{2}\sqrt{\dfrac{2\cdot 3^4\,\cancel{x^4}}{x^{\cancel{5}}}}=$

$=3\cdot 2\cdot x\sqrt{2x}-2\cdot 2^2\cdot 3\cdot x\sqrt{x}+\dfrac{3x^2 3^2}{2}\sqrt{\dfrac{1}{x}}=$

$=6x\sqrt{2x}-24x\sqrt{x}+\dfrac{27x^2}{2}\sqrt{\dfrac{1}{x}}$

solución: $6x\sqrt{2x}-24x\sqrt{x}+\dfrac{27x^2}{2}\sqrt{\dfrac{1}{x}}$

5. Realiza las operaciones siguientes y simplifica, si se puede:

a) $(6\sqrt{5}-3\sqrt{2})\cdot(2\sqrt{8}-\sqrt{3})$

$6\cdot 2\sqrt{5\cdot 8}-6\sqrt{5\cdot 3}-3\cdot 2\sqrt{2\cdot 8}+3\sqrt{3\cdot 2}=$

$=12\sqrt{40}-6\sqrt{15}-6\sqrt{16}+3\sqrt{6}=$

$=48\sqrt{10}-6\sqrt{15}-24\sqrt{2}+3\sqrt{6}$

solución: $48\sqrt{10}-6\sqrt{15}-24\sqrt{2}+3\sqrt{6}$

b) $(7\sqrt{3}+5\sqrt{7}-4\sqrt{5})(7\sqrt{5}-3\sqrt{7}+\sqrt{3})=$

$=7\cdot 7\sqrt{3\cdot 5}-7\cdot 3\sqrt{7\cdot 3}+7\sqrt{3\cdot 3}+5\cdot 7\sqrt{7\cdot 5}-$

$-5\cdot 3\sqrt{7\cdot 7}+5\sqrt{7\cdot 3}-4\cdot 7\sqrt{5\cdot 5}+4\cdot 3\sqrt{5\cdot 7}-$

$-4\sqrt{3\cdot 5}=49\sqrt{15}-21\sqrt{21}+7\cancel{\sqrt{3^2}}+35\sqrt{35}-$

$-15\cancel{\sqrt{7^2}}+5\sqrt{21}-28\cancel{\sqrt{7^2}}+12\sqrt{35}-4\sqrt{15}=$

$=\underbrace{49\sqrt{15}-4\sqrt{15}}_{45\sqrt{15}}\underbrace{-21\sqrt{21}+5\sqrt{21}}_{-16\sqrt{21}}+$

$+\underbrace{35\sqrt{35}+12\sqrt{35}}_{+47\sqrt{35}}+7\cdot 3-15\cdot 7-28\cdot 7=$

$+47\sqrt{35}\ +\ \underbrace{21-105-196}_{-280}$

$=-280+45\sqrt{15}-16\sqrt{21}+47\sqrt{35}$

solución: $-280+45\sqrt{15}-16\sqrt{21}+47\sqrt{35}$

c) $(2\sqrt{8}+5\sqrt{6}-3\sqrt{12})\cdot(4\sqrt{24}+5\sqrt{2}-7\sqrt{48})=$

$=(2\cdot 2\sqrt{2}+5\sqrt{6}-3\cdot 2\sqrt{3})\cdot$

$\cdot(4\cdot 2\sqrt{2\cdot 3}+5\sqrt{2}-7\cdot 2^2\sqrt{3})=$

$=(4\sqrt{2}+5\sqrt{6}-6\sqrt{3})\cdot(8\sqrt{6}+5\sqrt{2}-28\sqrt{3})=$

$=4\cdot 8\sqrt{2\cdot 6}+5\cdot 4\sqrt{2\cdot 2}-28\cdot 4\sqrt{2\cdot 3}+$

$+5\cdot 8\sqrt{6\cdot 6}+5\cdot 5\sqrt{6\cdot 2}-5\cdot 28\sqrt{6}-$

$-6\cdot 8\sqrt{3\cdot 6}-6\cdot 5\sqrt{3\cdot 2}+6\cdot 28\sqrt{3\cdot 3}=$

$=32\sqrt{12}+20\cancel{\sqrt{2^2}}-112\sqrt{6}+40\cancel{\sqrt{6^2}}+$

$+25\sqrt{12}-140\sqrt{18}-48\sqrt{18}-30\sqrt{6}+168\cancel{\sqrt{3^2}}$

$8=2^3$

$12=2^2\cdot 3$

$24=2^3\cdot 3$

$48=2^4\cdot 3$

$(32+25)\sqrt{12}-(112+30)\sqrt{6}-(140+48)$

$\sqrt{18}+20\cdot 2+40\cdot 6+168\cdot 3=$

$=57\sqrt{12}-142\sqrt{6}-188\sqrt{18}+\underbrace{40+240+504}_{784}$

$=784+57\sqrt{12}-142\sqrt{6}-188\sqrt{18}$

solución: $784+57\sqrt{12}-142\sqrt{6}-188\sqrt{18}$

6. Racionaliza:

a) $\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{2\cdot\sqrt{3}}{(\cancel{\sqrt{3}})^{\cancel{2}}}=\dfrac{2\sqrt{3}}{3}$

solución: $\dfrac{2\sqrt{3}}{3}$

b) $\dfrac{6}{\sqrt[3]{7}}\cdot\dfrac{\sqrt[3]{7^2}}{\sqrt[3]{7^2}}=\dfrac{6\sqrt[3]{7^2}}{\sqrt[\cancel{3}]{7^{\cancel{3}}}}=\dfrac{6\sqrt[3]{49}}{7}$

solución: $\dfrac{6\sqrt[3]{49}}{7}$

c) $\dfrac{\sqrt{8b^5}}{\sqrt{64b^3}}=\dfrac{\cancel{2}b^{\cancel{2}}\sqrt{2b}}{2^{\cancel{3}2}\cancel{b}\sqrt{b}}=\dfrac{b\sqrt{2b}}{2^2\sqrt{b}}\cdot\dfrac{\sqrt{b}}{\sqrt{b}}=\dfrac{b\sqrt{2b}\cdot\sqrt{b}}{2^2\sqrt{b}\sqrt{b}}$

$=\dfrac{b\sqrt{2b\cdot b}}{2^2\cancel{\sqrt{b^2}}}=\dfrac{\cancel{b}\sqrt{2b^2}}{4\cancel{b}}=\dfrac{b\sqrt{2}}{4}$

solución: $\dfrac{b\sqrt{2}}{4}$

d) $\dfrac{5}{3-\sqrt{3}}\cdot\dfrac{3+\sqrt{3}}{3+\sqrt{3}}=\dfrac{15+5\sqrt{3}}{3^2-(\cancel{\sqrt{3}})^{\cancel{2}}}=$

suma por diferencia

$(a+b)(a-b)=a^2-b^2$

$=\dfrac{15+5\sqrt{3}}{9-3}=\dfrac{15+5\sqrt{3}}{6}$

solución: $\dfrac{15+5\sqrt{3}}{6}$

e) $\dfrac{2}{-1+\sqrt{3}}\cdot\dfrac{-1-\sqrt{3}}{-1-\sqrt{3}}=\dfrac{-2-2\sqrt{3}}{(-1)^2-(\cancel{\sqrt{3}})^{\cancel{2}}}=\dfrac{-2-2\sqrt{3}}{1-3}=$

$=\dfrac{-2-2\sqrt{3}}{-2}=\dfrac{\cancel{-}2}{\cancel{-}2}+\dfrac{\cancel{-}2\sqrt{3}}{\cancel{-}2}=1+\sqrt{3}$

solución: $1+\sqrt{3}$

f) $\dfrac{3\sqrt{6}}{-\sqrt{5}+\sqrt{2}}\cdot\dfrac{-\sqrt{5}-\sqrt{2}}{-\sqrt{5}-\sqrt{2}}=\dfrac{-3\sqrt{6\cdot5}-3\sqrt{6\cdot2}}{(-\cancel{\sqrt{5}})^{\cancel{2}}-(-\cancel{\sqrt{2}})^{\cancel{2}}}$

$=\dfrac{-3\sqrt{30}-3\sqrt{12}}{5-2}=\dfrac{-3\sqrt{30}}{3}=\dfrac{-3\sqrt{30}}{3}+$

$+\dfrac{-3\sqrt{30}}{3}+\dfrac{-3\sqrt{12}}{3}=-\sqrt{30}-\sqrt{12}$

solución: $-\sqrt{30}-\sqrt{12}$

7. Racionaliza las siguientes operaciones y calcula el resultado, si es posible:

a) $\dfrac{3}{\sqrt{2}+1}-\dfrac{4}{\sqrt{2}-1}+\sqrt{3}=$

$=3\sqrt{2}-3-(4\sqrt{2}+4)+\sqrt{3}=$

$\dfrac{3}{\sqrt{2}+1}\cdot\dfrac{\sqrt{2}-1}{\sqrt{2}-1}=\dfrac{3\sqrt{2}-3}{(\cancel{\sqrt{2}})^{\cancel{2}}-1^2}=$

$=\dfrac{3\sqrt{2}-3}{2-1}=3\sqrt{2}-3$

$\dfrac{4}{\sqrt{2}-1}\cdot\dfrac{\sqrt{2}+1}{\sqrt{2}+1}=\dfrac{4\sqrt{2}+4}{(\cancel{\sqrt{2}})^{\cancel{2}}-1^2}=$

$=\dfrac{4\sqrt{2}+4}{2-1}=4\sqrt{2}+4$

$=3\sqrt{2}-3-4\sqrt{2}-4+\sqrt{3}=-\sqrt{2}-7+\sqrt{3}$

$=-7-\sqrt{2}+\sqrt{3}$

solución: $-7-\sqrt{2}+\sqrt{3}$

b) $\dfrac{2}{\sqrt{3}-1}-\dfrac{1}{1+\sqrt{2}}-\dfrac{1}{\sqrt{3}+\sqrt{2}}=$

$=\sqrt{3}+1-(-1+\sqrt{2})-(\sqrt{3}-\sqrt{2})$

$\dfrac{2}{\sqrt{3}-1}-\dfrac{\sqrt{3}+1}{\sqrt{3}+1}-\dfrac{2\sqrt{3}+2}{(\cancel{\sqrt{3}})^{\cancel{2}}-1^2}=$

$\dfrac{2\sqrt{3}+2}{3-1}=\dfrac{2\sqrt{3}+2}{2}=\sqrt{3}+1$

$\dfrac{1}{1+\sqrt{2}}\cdot\dfrac{1-\sqrt{2}}{1-\sqrt{2}}=\dfrac{1-\sqrt{2}}{1^2-(\cancel{\sqrt{2}})^{\cancel{2}}}=$

$=\dfrac{1-\sqrt{2}}{1-\sqrt{2}}=\dfrac{1-\sqrt{2}}{-1}=-1+\sqrt{2}$

$\dfrac{1}{\sqrt{3}+\sqrt{2}}\cdot\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=$

$=\dfrac{\sqrt{3}-\sqrt{2}}{(\cancel{\sqrt{3}})^{\cancel{2}}-(\cancel{\sqrt{2}})^{\cancel{2}}}=$

$=\dfrac{\sqrt{3}-\sqrt{2}}{3-2}=\sqrt{3}-\sqrt{2}$

$=\cancel{\sqrt{3}}+1+1-\cancel{\sqrt{2}}-\cancel{\sqrt{3}}+\cancel{\sqrt{2}}=2$

solución: $\cancel{\sqrt{3}}+1+1-\cancel{\sqrt{2}}-\cancel{\sqrt{3}}+\cancel{\sqrt{2}}=2$

c) $\dfrac{\sqrt{3}+\sqrt{6}}{\sqrt{6}-\sqrt{5}}-\dfrac{2}{\sqrt{6}-\sqrt{15}}+\dfrac{8}{\sqrt{6}}=$

$=\dfrac{\sqrt{3}+\sqrt{6}}{\sqrt{6}-\sqrt{5}}\cdot\dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=$

$=\dfrac{\sqrt{3\cdot 6}+\sqrt{3\cdot 5}+\sqrt{6\cdot 6}+\sqrt{6\cdot 5}}{(\cancel{\sqrt{}}6)^{\cancel{2}}-(\cancel{\sqrt{}}5)^{\cancel{2}}}=$

$=\dfrac{\sqrt{18}+\sqrt{15}+6+\sqrt{30}}{6-5}=$

$=\sqrt{18}+\sqrt{15}+6+\sqrt{30}$

$\dfrac{2}{\sqrt{6}-\sqrt{15}}\cdot\dfrac{\sqrt{6}+\sqrt{15}}{\sqrt{6}+\sqrt{15}}=$

$=\dfrac{2\sqrt{6}+2\sqrt{15}}{(\cancel{\sqrt{}}6)^{\cancel{2}}-(\cancel{\sqrt{}}15)^{\cancel{2}}}=\dfrac{2\sqrt{6}+2\sqrt{15}}{6-15}=$

$=\dfrac{2\sqrt{6}+2\sqrt{15}}{-9}$

$\dfrac{8}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}=\dfrac{8\sqrt{6}}{(\cancel{\sqrt{}}6)^{\cancel{2}}}=\dfrac{8\sqrt{6}}{6}=\dfrac{4\sqrt{6}}{3}=$

$=\sqrt{18}+\sqrt{15}+6+\sqrt{30}-\left(\dfrac{2\sqrt{6}+2\sqrt{15}}{-9}\right)+\dfrac{4\sqrt{6}}{3}=$

$-\cdot-=+$

$=\sqrt{18}+\sqrt{15}+6+\sqrt{30}+\dfrac{2\sqrt{6}}{9}+\dfrac{2\sqrt{15}}{9}+\dfrac{4\sqrt{6}}{3}$

$=\dfrac{9\sqrt{18}+9\sqrt{15}+54+9\sqrt{30}+2\sqrt{6}+2\sqrt{15}+12\sqrt{6}}{9}=$

$=\dfrac{54+9\sqrt{18}+11\sqrt{15}+14\sqrt{2}+9\sqrt{30}}{9}$

solución: $\dfrac{54+9\sqrt{18}+11\sqrt{15}+14\sqrt{2}+9\sqrt{30}}{9}$

8. Simplifica al máximo las siguientes expresiones logarítmicas:

a) $\lg a^6-\lg 100=6\lg a-4=$

$=6\lg a-\lg 2^2\cdot 5^2=6\lg a-2^2\underbrace{\lg 10}_{1}=$

$=6\lg a-4$

b) $\lg_3 81+\lg_3\dfrac{2}{9}=\lg_3 3^4+(\lg_3 2-\lg_3 9)=$

$=4\underbrace{\lg_3 3}_{1}+(\lg_3 2-2\underbrace{\lg_3 3}_{1})=4\cdot 1+\lg_3 2-2\cdot 1$

$=4+\lg_3 2-2=2+\lg_3{}^2$

solución: $2+\lg_3{}^2$

c) $\lg_2 64b^5-\lg_2\dfrac{144}{32b^3}$

$64=2^6$

$\lg_2 2^6b^5=\lg_2 2^6+\lg_2 b^5=6\lg_2 2+5\lg_2 b=$

$6\cdot 1+5\lg_2 b=6+5\lg_2 b$

$144=12^2\quad;\quad 32=2^5$

$\lg_2\dfrac{12^2}{2^5b^3}=\lg_2 12^2-\underbrace{\lg_2 2^5b^3}_{\lg_2 2^5+\lg_2 b^3}=$

$=\lg_2 2^5+\lg_2 b^3$

$=2\lg_2 12-5\underbrace{\lg_2}_{1}2+3\lg_2 b=$

$\lg_2 2^2\cdot 3=\lg_2 2^2+\lg_2 3=2\underbrace{\lg_2}_{1}2+\lg_2 3$

$=2\cdot 1+\lg_2 3=2+\lg_2 3$

$=2+\lg_2 3-5+3\lg_2 b=-3+\lg_2 3+3\lg_2 b$

solución: $-3+\lg_2 3+3\lg_2 b$

d) $\ln 7x^2+\ln 5x^3$

$(\ln 7+\ln x^2)+(\ln 5+\ln x^3)=$

$=\ln 7+2\ln x+\ln 5+3\ln x$

$=\underline{\ln 7+\ln 5}+5\ln x=$

$\ln 7=1,95$

$\ln 5=1,61$

$=1,95+1,61+5\ln x=$

$=3,56+5\ln x$

e) $\ln(b^7\cdot c^5)=\ln b^7+\ln c^5=7\ln b+5\ln c$

solución: $7\ln b+5\ln c$

f) $\lg_5 125x^4-\lg_5 15\sqrt{x}+3\lg_5\dfrac{150}{3\sqrt[4]{x}}$

$\lg_5=125x^4=\lg_5 125+\lg_5 x^4=5\underbrace{\lg_5}_{1}5+4\lg_5 x$

$125=5^3$

$=5+4\lg_5 x$

$\lg_5 15\sqrt{x}=\lg_5 15+\lg_5\sqrt{x}=\lg_5 3\cdot 5+\lg_5 x^{\frac{1}{2}}=$

$15=3\cdot 5\qquad \sqrt{x}=x^{\frac{1}{2}}$

$=\lg_5 3+\underbrace{\lg_5}_{1}5+\dfrac{1}{2}\lg_5 x=1+\lg_5 3+\dfrac{1}{2}\lg_5 x$

$3\lg_5\dfrac{150}{3\sqrt[4]{x}}=3(\lg_5 150-\lg_5 3\sqrt[4]{x})=$

$$3(\lg_5 2\cdot 3\cdot 5^2 - (\lg_5 3 + \lg_5 \sqrt[4]{x})) =$$

$$150 = 2\cdot 3\cdot 5^2 \qquad \sqrt[4]{x} = x^{\frac{1}{4}}$$

$$= 3(\lg_5 2 + \lg_5 3 + \lg_5 5^2 - (\lg_5 3 + \lg_5 x^{\frac{1}{4}})) =$$

$$= 3(\lg_5 2 + \lg_5 3 + 2\underbrace{\lg_5 5}_{1} - (\lg_5 3 + \frac{1}{4}\lg_5 x)) =$$

$$= 3(\lg_5 2 + \cancel{\lg_5 3} + 2 - \cancel{\lg_5 3} - \frac{1}{4}\lg_5 x) =$$

$$= 6 + 3\lg_5 2 - \frac{3}{4}\lg_5 x$$

$$5 + 4\lg_5 x - 1 - \lg_5 3 + \frac{1}{2}\lg_5 x + 6 + 3\lg_5 2 - \frac{3}{4}\lg_5 x$$

$$= 10 - \lg_5 3 + 3\lg_5 2 + \left(4 + \frac{1}{2} - \frac{3}{4}\right)\lg_5 x =$$

$$\frac{16 + 2 - 3}{4} = \frac{15}{4}\lg_5 x$$

$$= 10 - \lg_5 3 + 3\lg_5 2 + \frac{15}{4}\lg_5 x$$

solución: $10 - \lg_5 3 + 3\lg_5 2 + \frac{15}{4}\lg_5 x$

9. Expresa en base decimal y resuelve los siguientes logaritmos:

a) $\lg_3 17 = \dfrac{\lg 17}{\lg 3} = 2,58$

b) $\lg_5 4^3 = 3\lg_5 4 = 3\dfrac{\lg 4}{\lg 5} = 3\cdot(0,86) = 2,58$

c) $\lg_{\frac{1}{4}} 64 = \dfrac{\lg 64}{\lg \frac{1}{4}} = \dfrac{1,81}{-0,6} = -3,01$

d) $\lg_8 81 = \dfrac{\lg 81}{\lg 8} = 2,11$

10. Expresa en base natural y resuelve los siguientes logaritmos:

a) $\lg_6 7^4 = 4\lg_6 7 = 4\cdot\dfrac{\ln 7}{\ln 6} = 4\cdot 1,09 = 4,34$

b) $\lg_4 15 = \dfrac{\ln 15}{\ln 4} = 1,95$

c) $\lg_3 14 = \dfrac{\ln 14}{\ln 3} = 2,4$

d) $\lg_5 300 = \dfrac{\ln 300}{\ln 5} = 3,54$

e) $\lg_{\frac{1}{4}} 85 = \dfrac{\ln 85}{\ln \frac{1}{4}} = \dfrac{4,44}{-1,38} = -3,2$

11. Representa en notación científica:

a) $37325400000000 \rightarrow 3,73254\cdot 10^{13}$

b) 43 diezmillonésimas $\rightarrow 0,0000043 \rightarrow 4,3\cdot 10^{-6}$

c) $0,0000000738\cdot 10^{19} \rightarrow 7,38\cdot 10^{11}$

d) $375,93\cdot 10^{-2} \rightarrow 3,7593$

e) $4312,89\cdot 10^{-5} \rightarrow 0,0431289$

f) 3,5 diezmilésimas $\rightarrow 0,00035 \rightarrow 3,5\cdot 10^{-3}$

g) 1,57 billones $\rightarrow 1,57\cdot 10^{9}$